GEORGE WHITE
JUNIOR SCHOOL

The George White Junior School Calculations Policy

September 2019

What does maths look like in The North Norwich Cluster?

- To learn and make progress in mathematics, children need to be provided with a rich mixture of language, pictures and experiences to enable them to form their own understanding of the subject. In other words, they need to play with, talk about and see maths in many different ways to help them understand.

- In the North Norwich Cluster, we feel that maths is a subject that should be understood, rather than a series of procedures to be memorised. This is reflected in the planning and the content of maths lessons.

- The calculations policy should show the natural progression that a child should make through their mathematical education, it is not a year by year guide. Children might be working at different stages of the progression in different year groups, this shows that children learn at different speeds and in different ways. This is the advantage of learning maths in a progressive way, as it gives everyone time to make their own understanding of the subject.

- Written methods that are taught should help children to form their own understanding which, in time, will support future mental calculation. Some of the ‘traditional’ calculations strategies that have been taught in schools over the years do not allow children to ‘see’ the maths that they are doing, which means that the children are prone to make mistakes and lose confidence in their mathematical abilities. The methods in this document enable the maths to be seen, so that children can show their thinking, check their calculations and truly understand the maths that they are doing.

- This policy will show **how** we calculate, not **what** we calculate in The Inclusive Schools Trust.

Children's Progression of Understanding

Number Operation	Children's School Year	Phase of the Calculations Policy	Comments
Addition	N, R, 1, 2	Phase 1	Children in Key Stage 1 need to develop a good understanding of place value and to experience addition in many different forms. Those children who are more confident will start to work in Phase 2 towards the end of Year 1.
Addition	2, 3, 4, 5, 6	Phase 2	Children will develop their understanding of addition as they work within Phase 2. It will be necessary at times for a teacher to draw on the basic concepts in Phase 1 to support a child's understanding.
Subtraction	N, R, 1, 2	Phase 1	Children need to understand the concepts of 'take away' and 'difference'. Through use of objects and comparing lengths to numbers on a number line, children move to more abstract models.
Subtraction	2, 3, 4, 5, 6	Phase 2	Children will show subtraction in various ways with the numberline as the primary model for understanding. Use of objects such as beadstrings and counters will support the children's understanding. Models from Phase 1 will help with this.
Multiplication	N, R, 1, 2	Phase 1	The use of correct language is very important at this stage. As is the use of objects to count in groups and make into small arrays. Links with the numberline should be made through beadstrings.
Multiplication	2, 3, 4, 5, 6	Phase 2	Children build on their understanding from Phase 1 to create more complex arrays and make the link to the grid method. Use of times tables will help children to use the numberline method too. Children without a secure understanding of tables will be taught at Phase 1 to support heir understanding.
Division	N, R, 1, 2	Phase 1	The link with multiplication is key here, as is language too. Children need to do a lot of sharing and grouping of objects to secure their understanding.
Division	2, 3, 4, 5, 6	Phase 2	Once the child has a secure understanding of Phase 1, they can apply their times tables knowledge to Phase 2. Any gaps in their understanding will need to be addressed at Phase 1.

Models and Images for Understanding

Addition

Addition Rationale

Refer to commutative law to represent addition calculations:
a + b = b + a

Phase 1

Focus on combining sets of objects in practical ways. **Count all**, and then **count on**. Make connections between counting with objects and the number line/number track.

Count all – counting the first set one-by-one then continue counting second set.
Count on – remember the size of the first set and count on as you add the second set.

As the children become more secure with the number line method, they will start to use jumps of differing sizes. The recording of this will need to be adapted to their chosen size of jump.

Phase 2

Start with partitioning numbers using Base 10, Cuisenaire Rods or Numicon into tens and units first, then record as numbers i.e. 53=50+3. Then use this knowledge to represent it on a number line. Finally, do it numerically.
Once the children have an understanding of place value and partitioning, you can add by partitioning on the number line.
Children will then be introduced to the informal pencil and paper methods that will build on their existing mental strategies.

Formal written methods introduced in upper Key Stage 2 when children are secure in their understanding of place value, addition and consistent when using the number line strategies (see appendix).

Know More

- Understanding Mathematics for Young Children by Derek Haylock and Anne Cockburn – chapters 2 and 3.
- The Elephant in the Classroom: Helping Children Learn and Love Maths by Jo Boaler – chapter 7.

Vocabulary

Definition – addition is combining two or more numbers together to make a new number called the sum. The sign for addition is +. This is called the plus sign.

4 + 7 = 11

The **sum** of 4 and 7 is 11.
Four **plus** seven equals eleven.
Four **add** seven equals eleven.
7 **added** to 4 makes 11.
The **total** of 4 **and** 7 is 11.
Adding 7 to 4 **totals** 11.
What's 4 **more** than 7?
What is 4 **and** 7 **altogether**?
4 is one **part** and 7 is another **part**. What is your **whole?**

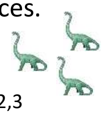
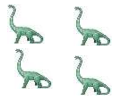
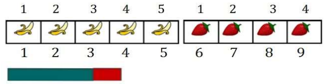
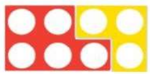
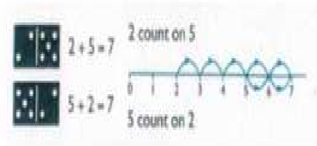
Links to subtraction

- Refer to subtraction as inverse of addition
- Use inverse to check subtraction calculations.

Models and Images for Understanding

Addition

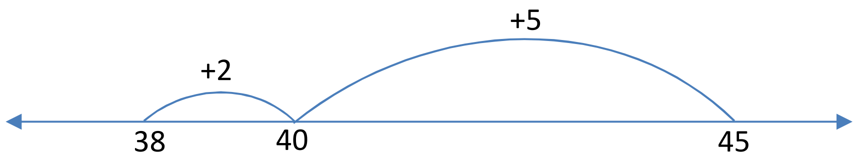
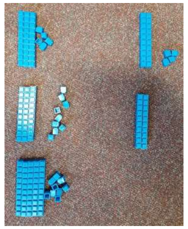
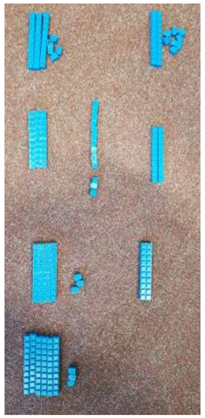
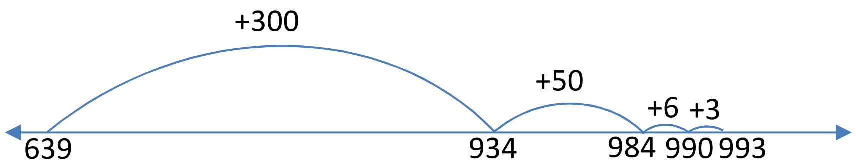
Addition - Phase 1

	Concrete	Pictorial	Abstract
Use addition as combining groups (aggregation).	Counting using familiar objects and resources.   1,2,3 1,2,3, 4 1,2,3, 4, 5, 6, 7	Introduce bar model representation. 	Using number sentences and beginning to calculate mentally. 7 + 2 = 9 2 + 7 = 9 9 = 2 + 7 9 = 7 + 2
Addition as counting on (augmentation).	**Count all**, then **count on** using familiar objects and resources. Part/whole model using objects.   4 1,2,3 4, 5, 6, 7   	Introduce number line. • Start with 2, then count up 5 more to reach the total. • Or • Start with 5, then count up 2 to reach the total.  As the children become more secure with the number line method, they will start to use jumps of differing sizes. Use part/whole model but draw objects instead.	

Models and Images for Understanding

Addition

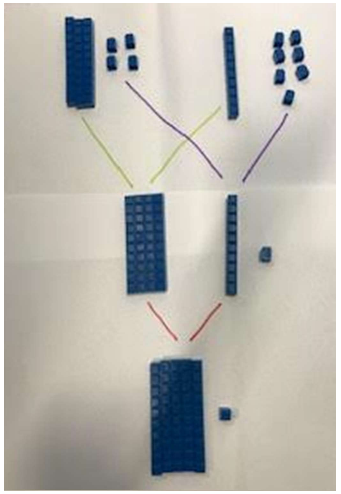
Addition - Phase 2

	Concrete	**Pictorial**
Bridging through ten	Use bead-strings to count on using bridging through ten. • Add on to the next whole ten. • Add remaining beads 	38 + 7 = 45 
Add using partitioning (augmentation)	Base ten to count on using partitioning. Straight forward adding on using partitioning: • Add on the tens • Add on the ones   Adding using exchanging: • Combine ones together • Exchange 10 ones for a ten and add to the ten place value column • Add the remaining tens	Place value counters should be available to support understanding. 359 + 634 = 993 

Models and Images for Understanding

Addition

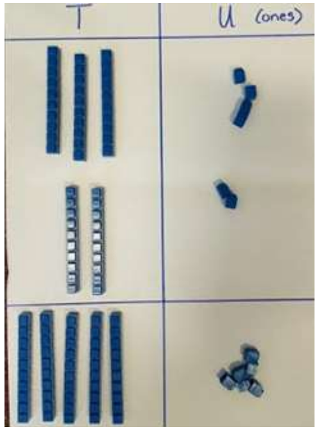
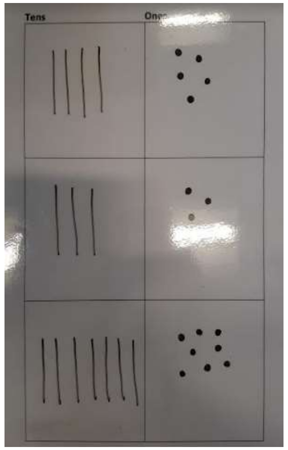
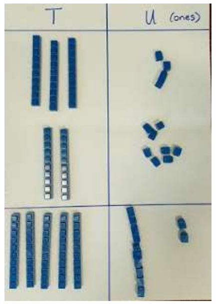
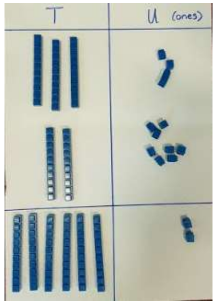
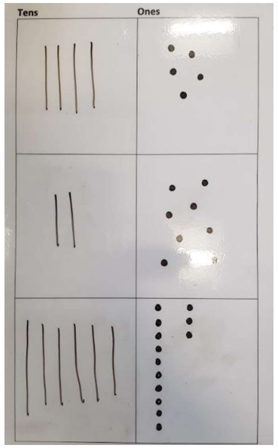
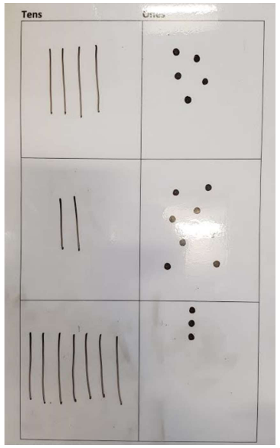
Addition - Phase 2

	Concrete	Pictorial	Abstract
Bow Tie method		34 + 17 = 40 + 11 = 51 125 + 213 = 300 + 30 + 8 = 338	43 + 24 = 40 + 20 = 60 3 + 4 =7 60 + 7 = 67

Models and Images for Understanding

Addition

Addition - Phase 2

	Concrete	**Pictorial**
Introducing column addition	 • Use place value mat to show understanding. • Combine Base 10 for each place value column	
	Addition using exchanging: • Combine ones and exchange 10 for a whole ten to add to tens place value column  	 

Models and Images for Understanding
Subtraction

Subtraction Rationale

It is essential that children are exposed to each of 6 structures of subtraction throughout their education.

- Partitioning and taking away (This is not a hierarchy)
- Comparison (difference)
- Finding the complement
- Counting back
- The inverse of addition.
- Bridging down through 10

Phase 1

Children should be taught to compare from Nursery age, so that they gain a sense of the relative size of numbers and amounts. From there, they identify the difference between amounts and take away one from the other. These should both be taught at the first point of using subtraction, with a strong emphasis on the language being used. Once the children are aware that one number is bigger than another, explore ways of finding the difference between them (see 6 structures above).

Phase 2

Emphasis on the use of the number line to support understanding of finding the difference and counting back.

All subtraction calculations can be worked out accurately by using the number line method. The method also supports the development of number sense, which can then be applied to different contexts in written calculation as well as in mental calculation too.

Children are not encouraged to use decomposition for subtraction calculations unless they have shown to have a relational understanding of subtraction and place value. If decomposition is taught without this knowledge, misconceptions can be developed and mistakes made. Please see the subject leader for maths for further guidance. (Also see the Appendix to this document)

Know More

- Understanding Mathematics for Young Children by Derek Haylock and Anne Cockburn – chapters 2 and 3.
- The Elephant in the Classroom: Helping Children Learn and Love Maths by Jo Boaler – chapter 7.

Vocabulary

Definition:

1. Subtraction is taking away one number from another.
2. Subtraction is the difference between two numbers.
3. Subtraction is the inverse of addition.
4. The sign for subtraction is -. This is called the minus sign.

6 – 2 = 4

6 **minus** 2 equals 4.
Find the difference between 6 and 2.
Six **subtract** two is equal to four.
Takeaway 2 from 6.
What is 2 **less than** 6?
How **much more** is 6 from 2?
There are **more** red cubes **than** blue cubes.

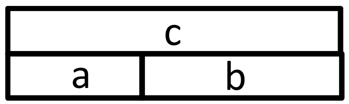
Links to addition

- Refer to subtraction as inverse of addition
- Use inverse to check subtraction calculations.

Models and Images for Understanding

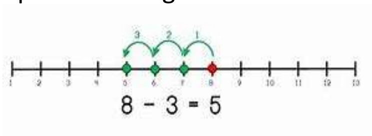
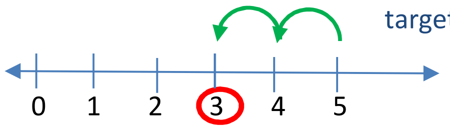
Subtraction

Subtraction - Phase 1

	Concrete	Pictorial	Abstract
Comparing	Ask questions to start comparing concrete and pictorial representations: • What's the same? What's different? • Which snake is **longer**? Which is **shorter**? • What do you notice? • Arrange the coins in lines to make the link between the coins lined up and numbers on a number track. 9 is further along the track than 6.  		8 – 5 = 3 8 – 3 = 5 5 = 8 – 3 3 = 8 – 5
Inverse of addition	This method relies on the children having had opportunities for the development of numerical reasoning. Understanding can be created by encouraging the child to make connections between the written calculation, language and the physical acts of moving a number of beads along the bead string or jumps along the number line. To help to achieve this, use the following AfL questions and language: • ***If***, 7 + 4 = 11, what might 11 – 4 = ? • What do you notice about the number sentences 7+4=11 and 11-4=7 ? • What's the same and what's different about them? • Can you think of another number sentence that uses the numbers 11, 4 and 7? • Can you write down the whole 'calculation family' for 7 + 4 = 11?  a + b = c c – a = b c – b = a		

Models and Images for Understanding
Subtraction

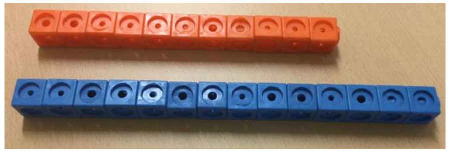
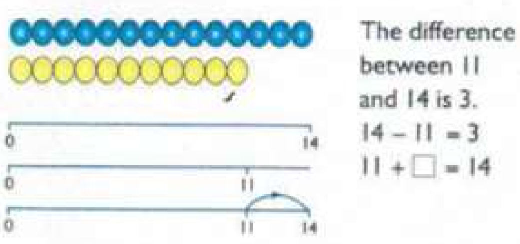
Subtraction - Phase 1

	Concrete	Pictorial	Abstract
Counting back/away	In each case, link the separation from the whole with counting the beads on the bead strings as you take them away. 3 less than 8 means that you count back three beads. Start with 8 and move three away. 	Counting in ones on a number line. As the children become more secure with the number line method, they will start to use jumps of differing sizes.  At the same time as using the beadstring, count back 3 numbers along the number line from 8 to 5.	5 - ____ = 3
Counting back to (missing numbers)	Start with the first number, identify the target number and count how many beads you need to take away. 5 - ____ = 3 Count back until you reach 3. How many have you subtracted? 	Link this to identifying the numbers on a number line and counting back to the target number. Count back 2 to reach the target of 3. 	

Models and Images for Understanding

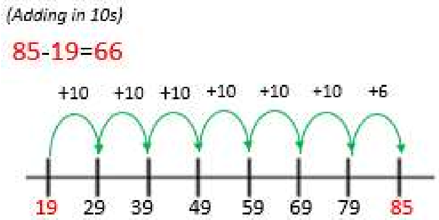
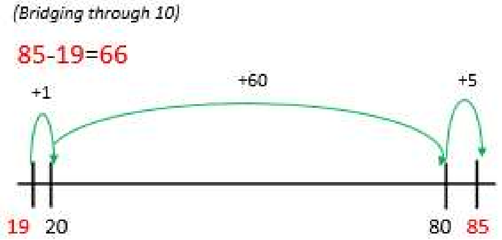
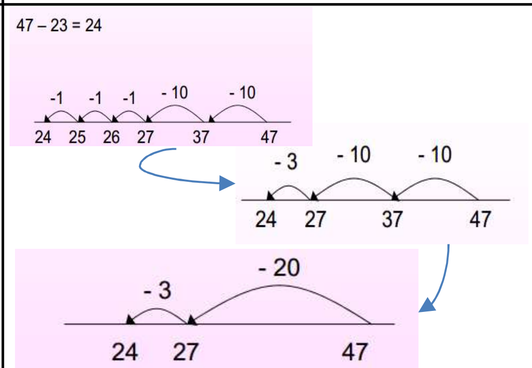
Subtraction

Subtraction - Phase 1

	Concrete	Pictorial	Abstract
Finding the difference (counting up)	Exemplifies the gap in the number line between the two numbers of the calculation. Avoid the idea that “you put the small number at one end and the big number at the other”. Focus on the relative size of the numbers and identifying the ‘difference’ between them. Use beadstrings, multilink or numicon to show the difference between two numbers. 	Use numberlines to show the difference between two numbers then count up. 	

Models and Images for Understanding
Subtraction

Subtraction - Phase 2

	Concrete	Pictorial	Abstract
Finding the difference (counting up in jumps)	Use beadstrings, numicon, multilink cubes to enable children to count up. Encourage counting up in jumps, using the bands of Colour. 	  The children should be encouraged to draw their number lines with the 'jumps' as representative in size as possible. Same method used with increasingly larger numbers.	85 – 19 = 66 85 – 66 = 19 19 = 85 – 66 66 = 85 – 19 **See subject leader and appendix for formal written methods.**
Counting back using place value	Identify the starting point then count beads away. Encourage children to count in jumps using the colour bands. 		

Models and Images for Understanding *Multiplication*

Multiplication Rationale

Four different ways of thinking about multiplication are:

- as repeated addition, for example 3 + 3 + 3 + 3
- as an array, for example four rows of three objects
- as a scaling factor, for example, making a line 3 cm long four times as long.
- as the inverse of division.

The language of Multiplication:

The use of the multiplication sign can cause difficulties. Strictly, 3 × 4 means four threes or 3 + 3 + 3 + 3. Read correctly, it means 3 multiplied by 4 (*or 3, 4 times*). **However, colloquially it is read as '3 times 4', which is 4 + 4 + 4 or three fours.** Fortunately, multiplication is commutative: 3 × 4 is equal to 4 × 3, so the outcome is the same.

Children's understanding of multiplication starts with unitary counting. Once they can count single objects, they should start to count in twos, fives and tens

Through a CPA approach the children will create their own internal representations of the multiplication facts.

Through the direct modelling with materials children should progress from unitary counting, counting on with use of their addition facts. Alongside this children should be provided with opportunities to count thematically and recognise number patterns and apply multiplication facts.

The repeated addition structure of multiplication can be easily represented on a number line. Initially, this should be represented on a number track which displays each number from 1 to 20.

To develop the concept of repeated addition, use the language of multiplication carefully. For the example above; 2+2+2+2+2=10 which is the same as 2x5=10, the language to use to aid conceptual development is: 2, 5 times.

An array could show 3x4 or 4x3. It doesn't matter whether you read it as a column of 3 dots, 4 times, or a row of 4 dots, 3 times. What is important is that the children see the commutativity.

Once children have experienced different ways of splitting up arrays, they should start to turn this into the conventional layout of the grid method showing the proportionality for each box within the grid.

Know More

'Teaching children to calculate mentally' (Department for Education, 2010)

Relationships and progression among addition and multiplication strategies (Thompson 2008)

Vocabulary

Definition – Multiplication is adding lots of the same number together. The sign for multiplication is x.

2 x 5 = 10

What is 5 **groups of** 2?
What is 2, five **times**?
What is 5 **sets of** 2?
Use **repeated addition** to find a total.
What is two **times** five?

Commutative
Array
Repeated Addition

Links to Division

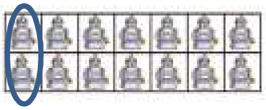
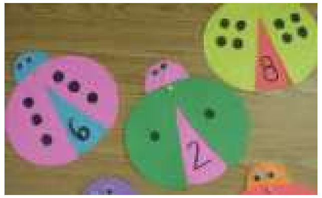
All of the models of multiplication can and should be used to develop children's understanding of division as the **inverse of** multiplication.

Links to Addition

Repeated addition

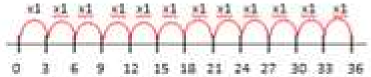
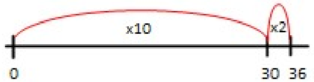
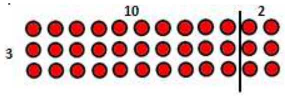
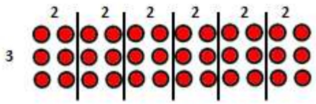
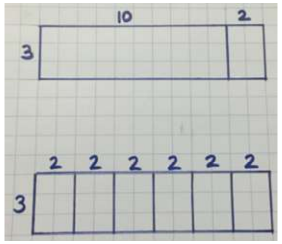
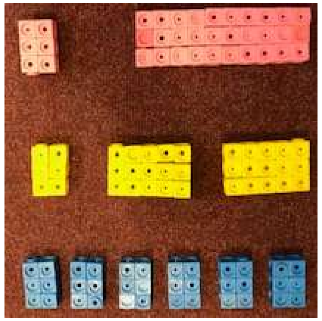
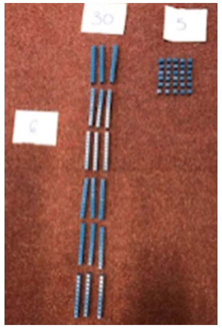
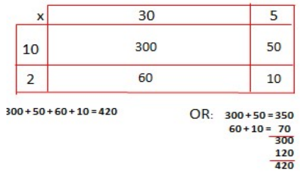
Models and Images for Understanding
Multiplication

Multiplication- Phase 1

	Concrete	**Pictorial**	**Abstract**
Multiplication as repeated addition.	Using familiar objects and resources find groups of with repeated addition. 2 x 3 = 6   2 + 2 + 2 = 6	2 + 2 + 2 = 6 2 x 3 = 6 	Using number sentences and beginning to calculate mentally.  2 + 2 + 2 = 6 2 x 3 = 6
Represent repeated addition as an array.	2x5  5x2	Understand the visual representation (link to tables) 	
Doubling numbers within 20 (link to halving)	Use familiar objects and resources 	Use a variety of models and images  	1x2 2x2 3x2 4x2 5x2 6x2 7x2 8x2 9x2 10x2 Doubles 1 + 1 = 2 + 2 = 3 + 3 = 4 + 4 = 5 + 5 = 6 + 6 = 7 + 7 = 8 + 8 = 9 + 9 = 10 + 10 =

Models and Images for Understanding
Multiplication

Multiplication- Phase 2

	Concrete	Pictorial	Abstract
Multiplication using a number line with varying multiples to increase the fluency.	3 x 12 =36  As children become more confident they would increase jump size, 3x10, 3x2	3 x 12 =36   	3 x 12 = 36 12 x 3 = 36 .
Represent multiplication as an array	Using multiples of 10 is a preferred method, but it is not the o  3 x 12 =36  	 	
Link arrays to grid method	  Make arrays, divide them up to calculate.	 Before calculating, the children should make an estimate of the anticipated answer. Ensure that all grid boxes are proportional to their value.	35 x 12 = 420 *See appendix for formal methods of multiplication (Year 4 and 5 only)

Models and Images for Understanding

Division

Division Rationale

Three different ways of thinking about division are:

- as sharing
- as grouping
- as the inverse of multiplication
- halving

The language of Division:

Sharing is often taught in the earlier years with small numbers and can be calculated easily with objects that can be manipulated which progress to other methods as the numbers increase.

Using grouping with objects, line the objects (bead string) up next to the numberline so that the children can see the connection between the line of objects and the numbers on the numberline. 6 ÷ 2 = 3. 6 divided into groups of 2. There are 3 groups.

Use other representations, such as bead strings, to support the understanding of division on the number line. Children can then use the Division Fact Box to list multiplication facts that can be used in the calculation, linking multiplication as the inverse of division.

To find half, children will first partition the whole number down into simple facts before halving each part and then combining to find the answer (56, 50 and 6, 20 and 20 and 10 and 6, 10+10+5+3=28)

Drawing arrays to illustrate division can enable children to make the link with multiplication.

The number line for multiplication is very similar to division however the starting point differs.

The links between the two must be consistent and frequent within each lesson

Know More
Understanding Mathematics for Young Children (Haylock and Cockburn 2013)

Vocabulary

Definition – Division is sharing things equally. Division is grouping into sets of the same size. The sign for division is ÷

15÷5=

What is 15 divided into 5?

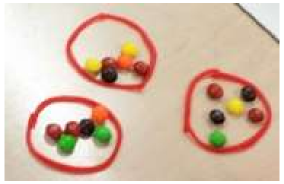
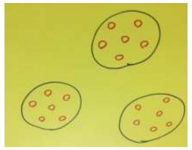
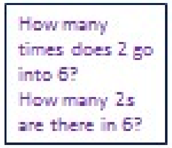
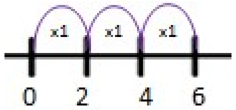
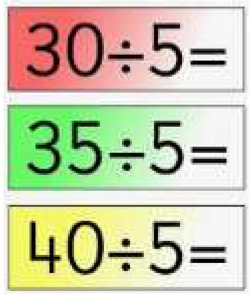
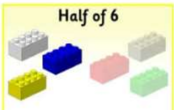
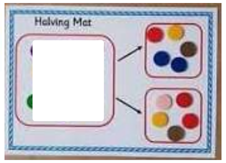
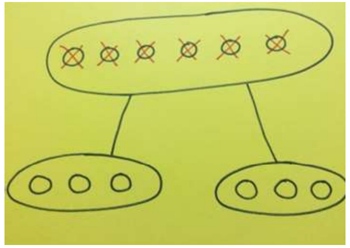
Divide
Share equally
Divide into
Groups of
Sets of

Links to Multiplication
All of the models of multiplication can and should be used to develop children's understanding of division as the **inverse of** multiplication.

Models and Images for Understanding

Division

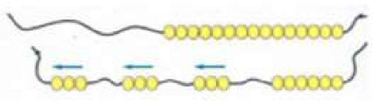
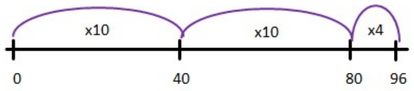
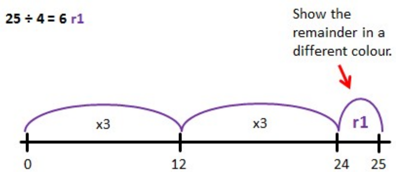
Division- Phase 1

	Concrete	Pictorial	Abstract
Division as sharing and grouping with objects	Use familiar objects to share equally into groups. 18 ÷ 3 = 6 	18 ÷ 3 = 6   6 ÷ 2 = 3 	
Halving using numbers within 20 (link to doubling)	Put objects into 2 equal groups.  6÷2=3 		

Models and Images for Understanding

Division

Division- Phase 2

	Concrete	Pictorial	Abstract
Division on a number line	Use representations, such as bead strings, to support the understanding of division on the number line. 	(Link to bead strin **Division Fact Box** List multiplication facts that can be used in the  **Fact Box** 4x10=40 4x20=80 4x5=20 4x2=8 4x4=16	30÷5= 35÷5= 40÷5=  * See appendix for Year 5 and Year 6
Division with remainders	25÷4=6 r1 	 *Note: Upper KS2 to convert remainders to fractions and decimals*	

Models and Images for Understanding

Problem Solving

Problem Solving Rationale

Problem solving can take many forms, they are not just word problems. These include:

- Word problems
- Problems which involve finding all possibilities
- Visual problems
- Logic problems
- Problems which involve spotting rules and patterns (From www.nrich.maths.org)

Every lesson should involve these sorts of activities which all help to develop children's problem solving skills. The skills to be developed are:

- Pattern spotting
- Working systematically
- Using diagrams and pictorial information
- Working backwards
- Trial and improvement
- Visualising
- Conjecturing and generalising
- Reasoning logically

The Bar Method

The Bar Method is not a calculating tool, rather it is a representation that shows the structure of a word problem. By revealing the structure, it is easier to **see** which parts of the problem are **known** and which parts are **unknown**. From this point, you can select the number operation(s) that you require and can solve the problem. This follows the Concrete-Pictorial-Abstract (CPA) model of conceptual development.

Know More

- www.nrich.maths.org
- Problem Solving and Reasoning. (Book) Rising Stars. Y1 to Y6.

Vocabulary

Key Questions to ask in lessons:

- What do you notice?
- What's the same? What's different?
- Can you find another, and another …
- What has stayed the same? What has changed?
- How do you know…
    - …that you've found all of the solutions?
    - …that it is correct?
    - …
- What do you know that will help you to work that out?
- What do you know and what don't you know?
- How will you find that out?

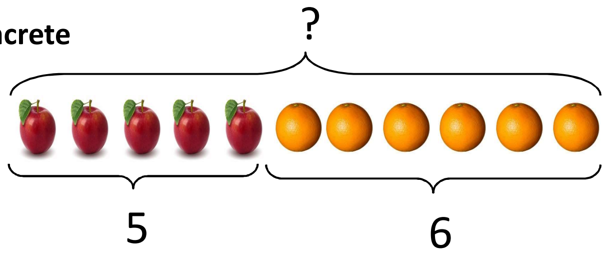
Models and Images for Understanding

Problem Solving

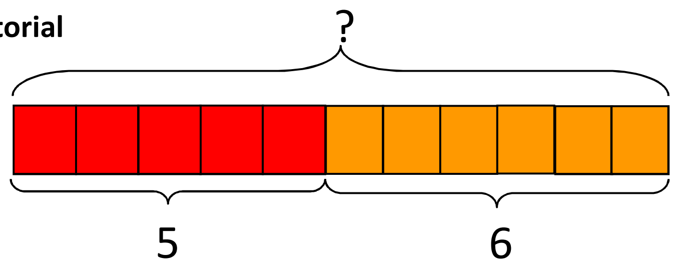
Part-whole model for addition and subtraction

There are 5 apples and 6 oranges. How many pieces of fruit altogether?

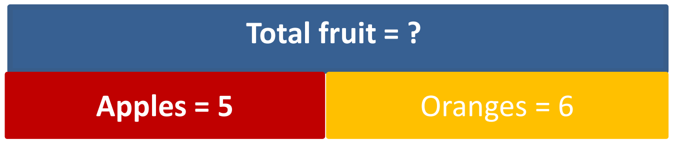
Concrete



Pictorial



Abstract



Apples + Oranges = Total fruit. 5+6=?
If the total was known but the oranges were not, then it would be: total fruit – apples = oranges.

The bar method can also be used to help solve problems relating to multiplication, division, fractions, ratio and proportion. In each case, the user needs to consider which quantities or relationships they know, and which quantities or relationships they don't know. Through representing each part with bars, they can then deduce the parts unknown and solve the problem.

In each case, the process should start with the concrete model before moving onto a pictorial representation and then finally by using an abstract representation in the form of a bar, or bars.

A powerpoint presentation is available that explains the bar method in more detail, as this document would not do it justice. Please see the subject leader for mathematics for further guidance.